

# Uncertainty of the Calibration of Paired Temperature Sensors for Heat Meters

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**Abstract** In heat meters, the temperatures of the incoming and the outgoing water are measured with calibrated platinum resistance thermometers. They are subject to legal metrology and the requirements for the uncertainty associated with the temperature difference are specified in national and regional standards. It is difficult to calculate this uncertainty, since it depends on the characteristics of both sensors and correlation needs to be considered. Causes of that correlation include calibration in the same facility, losses due to heat conduction, and common electrical measurements. Therefore, an analytical calculation of the temperature difference and the associated uncertainty is quite complicated, requiring several steps, corrections, and approximations. Monte Carlo techniques avoid these difficulties by simulating the calibration of both sensors simultaneously. As expected, the correlation reduces the uncertainty associated with the temperature difference indicated by the sensor pair considerably. Monte Carlo simulation and analytical calculations are briefly described and their results are compared.

**Keywords** Correlation · Heat meters · Legal metrology · Monte Carlo method · Paired sensors · Uncertainties

## 1 Introduction

Heating of houses and apartments is one of the largest areas of human energy consumption. To manage the energy distribution, it is necessary to measure the energy consumed

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by heating. In cases where the heat production (from coal, oil, or electrical energy) does not occur at the same location as the heat consumption, the heat flux has to be measured. This is the case for district heating, and also for the heating of larger buildings.

Heat is usually distributed by hot water. A typical heat meter consists of a pair of temperature sensors that measure the temperature of the incoming and the outgoing water, a flow meter that measures the amount of water going through the system, and a calculator that computes the heat dissipated in the system from the temperature difference between the incoming and the outgoing water and the volume of water; the absolute temperature of the water gives only a correction and can be less accurate than the temperature difference.

Verification or calibration of the heat-meter components is required for most applications by national regulations. The procedure and the acceptable tolerances are defined in international recommendation OIML R75 [1] and regional standards (e.g., EN 1434) [2]. This article deals only with the pair of temperature sensors that usually consist of two industrial platinum sensors (Pt-100, Pt-500, or Pt-1000). The calculation of the amount of heat, in many cases, is based on the assumption that both temperature sensors follow the resistance-temperature characteristic of the Callendar–van Dusen equation (CvD) [3], which for temperatures above 0°C is a simple quadratic equation:

$$R(t) = R_0(1 + At + Bt^2) = R_0 + R_0At + R_0Bt^2. \quad (1)$$

The task of the manufacturer or of special offices is to form pairs of sensors from a batch in such a way that for each pair the error in the calculation of the temperature difference  $\Delta t$  using the CvD equation with standard coefficients as defined in IEC 60751 [3] is smaller than the tolerance or the maximum permissible error (*MPE*). This error is called the mismatch of the pair. The *MPE* is defined as

$$MPE_{\text{rel}} = (0.5 + 3\Delta t_{\text{min}}/\Delta t) \% \quad \text{or} \quad MPE = 0.005\Delta t + 0.03\Delta t_{\text{min}}, \quad (2)$$

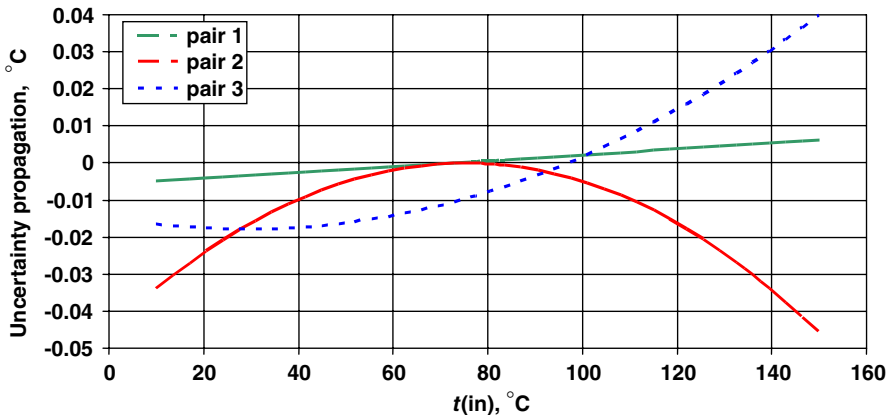
where  $\Delta t_{\text{min}}$  is the smallest temperature difference acceptable in the heating system, usually 3°C. So, the *MPE* for the pairing of the sensors is

$$MPE(\Delta t = 3^\circ\text{C}) = 0.105^\circ\text{C} \quad \text{and} \quad MPE(\Delta t = 40^\circ\text{C}) = 0.29^\circ\text{C}. \quad (3)$$

The statement that the mismatch is smaller than the *MPE* should be interpreted that the sum of the absolute value of the mismatch (also named matching error  $\Delta t_{\text{matching}}$ ) and the expanded uncertainty ( $k = 2$ ) is smaller than the *MPE*. The standards do not pose any requirements on the contribution of the matching error, but request merely that the expanded uncertainty of the calibration systems is  $<0.2MPE$ . Such requirements are quite usual in legal metrology, but they are not in agreement with the BIPM-ISO *Guide to the Expression of Uncertainty in Measurement* (GUM) [4]. In the following, a method for the analytical calculation of the uncertainty of the matching error will be discussed and compared to a Monte Carlo approach.

**Table 1** Parameter sets used for the calculation of the curves in Fig. 1

	$\delta t$ (in, 20°C)	$\delta t$ (out, 20°C)	$\delta t$ (in, 70°C)	$\delta t$ (out, 70°C)	$\delta t$ (in, 120°C)	$\delta t$ (out, 120°C)
Pair 1	0.01°C	0.01°C	0	0	0.01°C	0.01°C
Pair 2	-0.01°C	0.01°C	0	0	-0.01°C	0.01°C
Pair 3	-0.01°C	0.01°C	0	0.01°C	0.01°C	-0.01°C

**Fig. 1** Uncertainty propagation for  $\Delta t = t_{\text{in}} - t_{\text{out}} = 10^\circ\text{C}$  for temperature deviation  $\delta t$  in the calibration baths. The curves have been calculated for three pairs of sensors with the parameter sets given in Table 1

## 2 Calibration of Temperature Sensor Pairs

The temperature sensors are usually calibrated at three different temperatures in stirred liquid baths. From these calibrations, the constants of the CvD equation ( $R_0$ ,  $A$ , and  $B$ ) are calculated and sensors with similar constants are matched as a pair. The calculation of the matching error is quite complicated for the following reasons:

1. All sensors are calibrated in the same system; hence, many uncertainty contributions are correlated.
2. The matching error must be smaller than the *MPE* not only at the temperatures used for the calibration, but for a range of temperatures and temperature differences.

For a better understanding, a program for the propagation of uncertainties was developed. This program calculates the matching error for a given temperature difference  $\Delta t$  from the deviations of the sensors from the CvD equations with the constants defined in the standard IEC 60751. The deviation  $\delta t$  can be set for both sensors individually as temperature equivalents for the calibration temperatures of the stirred liquid baths (e.g., 20, 70, and 120°C). The temperature difference  $\Delta t$  can be chosen, and the matching error calculated for the temperature range of the incoming water (e.g., 10–150°C). An example of such a calculation using the values from Table 1 is shown in Fig. 1.

It is not surprising that a deviation of both sensors by the same amount leads to small matching errors (pair 1), while deviations in opposite directions lead to much

**Table 2** Uncertainty contributions in the calibration of the sensors

a	Description	Bath temperature			PDF <sup>b</sup>
		20°C	70°C	120°C	
$T_{Ho}$	Homogeneity of bath	1.73 mK	1.73 mK	1.39 mK	R, u
$T_S$	Stability of bath	1.15 mK	1.15 mK	0.92 mK	R, u
$T_{RS}$	Reference standards	4.30 mK	4.30 mK	6.90 mK	N, c
$T_{HC}$	Heat conduction	1.44 mK	1.44 mK	1.15 mK	R, c
$R_T$	Electrical effects, thermo-voltages	0.23 mΩ	0.23 mΩ	0.37 mΩ	R, u
$R_{EM}$	Electrical measurements	0.56 mΩ	0.56 mΩ	0.92 mΩ	N, c

<sup>a</sup> For all quantities  $T_x: ET_x = 0K$  and for all quantities  $R_x: ER_x = 0\Omega$ ;  $E$  is the operator for computing the expectation. The correction for the heat conduction is  $-\sqrt{3}u(T_H)$ !

<sup>b</sup> Probability distribution function PDF:Rectangular, Normal; u=uncorrelated, c=correlated

**Table 3** Calibration results for both sensors of the matched pair

Parameter	Thermometer 4002 (in)	Thermometer 4006 (out)
$R(20^\circ C)$	107.805 Ω	107.783 Ω
$R(70^\circ C)$	127.075 Ω	127.062 Ω
$R(120^\circ C)$	146.028 Ω	146.021 Ω

larger matching errors (pairs 2 and 3). Roughly speaking, if both sensors are shifted by the same deviation, the temperature difference may still be more or less correct.

Temperature shifts of both sensors in the same direction are just the effect of correlated uncertainty distributions. If a calibration bath has a systematic uncertainty, both sensors will be affected in the same way and the deviations will partly cancel when taking the temperature difference. So, the correlation of uncertainties may result in a smaller total uncertainty.

In the following, we will consider a matched pair of sensors calibrated at temperatures of 20, 70, and 120°C. The uncertainty components for the calibration are given in Table 2. It should be mentioned that these uncertainties are quite small for an industrial calibration laboratory and can only be achieved with considerable effort. The measurement results for the thermometers are given in Table 3.

The parameters ( $R_0$ ,  $R_0A$ , and  $R_0B$ ) for the CvD equation are the solutions of the equations,

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} = \begin{pmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ 1 & t_3 & t_3^2 \end{pmatrix} \begin{pmatrix} R_0 \\ R_0A \\ R_0B \end{pmatrix} \tag{4}$$

Using  $D = \begin{pmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ 1 & t_3 & t_3^2 \end{pmatrix}$ , the solution of the equation is

**Table 4** Parameters of the thermometers as calculated from the calibration results

Parameter	Thermometer 4002 (in)	Thermometer 4006 (out)
$R_0$	100.00824 $\Omega$	99.9818 $\Omega$
$R_0A$	0.391106 $^{\circ}\text{C}^{-1}$	0.391340 $^{\circ}\text{C}^{-1}$
$R_0B$	$-6.34 \times 10^{-5} \text{ } ^{\circ}\text{C}^{-2}$	$-6.40 \times 10^{-5} \text{ } ^{\circ}\text{C}^{-2}$

$$R_0 = \begin{vmatrix} R_1 & t_1 & t_1^2 \\ R_2 & t_2 & t_2^2 \\ R_3 & t_3 & t_3^2 \end{vmatrix} 1/D; \quad R_0A = \begin{vmatrix} 1 & R_1 & t_1^2 \\ 1 & R_2 & t_2^2 \\ 1 & R_3 & t_3^2 \end{vmatrix} 1/D; \quad R_0B = \begin{vmatrix} 1 & t_1 & R_1 \\ 1 & t_2 & R_2 \\ 1 & t_3 & R_3 \end{vmatrix} 1/D \quad (5)$$

The inversion is numerically stable since the value of  $|D|$  is about 25,000. The solutions for both thermometers considered are given in Table 4.

### 3 Approximation of the Uncertainties Using Analytical Techniques

In the following section, all uncertainty contributions are converted to uncertainties in resistance using the CvD equation. This means that all calibration uncertainties resulting from measurements in bath  $i$  are included in  $u(R_i)$ . The propagation of uncertainties can then be calculated using the derivatives:

$$\begin{aligned} \frac{\partial R}{\partial R_1} &= (1/D) \left\{ \begin{vmatrix} t_2 & t_2^2 \\ t_3 & t_3^2 \end{vmatrix} - t \begin{vmatrix} 1 & t_2^2 \\ 1 & t_3^2 \end{vmatrix} + t^2 \begin{vmatrix} 1 & t_2 \\ 1 & t_3 \end{vmatrix} \right\} \\ \frac{\partial R}{\partial R_2} &= (1/D) \left\{ - \begin{vmatrix} t_1 & t_1^2 \\ t_3 & t_3^2 \end{vmatrix} + t \begin{vmatrix} 1 & t_1^2 \\ 1 & t_3^2 \end{vmatrix} + t^2 \begin{vmatrix} 1 & t_1 \\ 1 & t_3 \end{vmatrix} \right\} \\ \frac{\partial R}{\partial R_3} &= (1/D) \left\{ \begin{vmatrix} t_1 & t_1^2 \\ t_2 & t_2^2 \end{vmatrix} - t \begin{vmatrix} 1 & t_1^2 \\ 1 & t_2^2 \end{vmatrix} + t^2 \begin{vmatrix} 1 & t_1 \\ 1 & t_2 \end{vmatrix} \right\} \end{aligned} \quad (6)$$

In the following, it is assumed that both thermometers have been calibrated using the same equipment and that there are only uncertainty contributions that are totally correlated (reference standards including drift, stability of the bath, etc.) and contributions that are totally uncorrelated (homogeneity and stability of bath temperatures, etc.). Uncertainty contributions from different calibration temperatures  $t_i$  are not correlated. As a rough approximation, the contributions from Table 2 have been summarized to only two contributions for each thermometer, the correlated contributions  $u_c(t_{\text{in}})$  and  $u_c(t_{\text{out}})$  and the uncorrelated contributions  $u_u(t_{\text{in}})$  and  $u_u(t_{\text{out}})$ . If the temperatures are converted to resistances using the sensitivity coefficient derived from the CvD equation and the uncertainties of the temperature values are included in the uncertainties of the resistances, the uncertainty of the temperature difference expressed as a difference of the resistances,  $\Delta R = R(t_{\text{in}}) - R(t_{\text{out}})$ , can be calculated as

$$u^2(\Delta R) = \left( u_c^2(R_{\text{in}} - R_{\text{out}}) \right) + u_u^2(R_{\text{in}}) + u_u^2(R_{\text{out}}) \quad (7)$$

or explicitly [4]:

$$\begin{aligned}
 u^2(\Delta R) &= \sum_{i=1}^3 \left( \frac{\partial R}{\partial R_{i,\text{in}}} u(R_{i,\text{in}}) \right)^2 + \sum_{i=1}^3 \left( \frac{\partial R}{\partial R_{i,\text{out}}} u(R_{i,\text{out}}) \right)^2 \\
 &+ 2 \sum_{i=1}^2 \sum_{j=i+1}^3 \frac{\partial R}{\partial R_{i,\text{in}}} u(R_{i,\text{in}}) r(R_{i,\text{in}}, R_{j,\text{in}}) \frac{\partial R}{\partial R_{j,\text{in}}} u(R_{j,\text{in}}) \\
 &+ 2 \sum_{i=1}^2 \sum_{j=i+1}^3 \frac{\partial R}{\partial R_{i,\text{out}}} u(R_{i,\text{out}}) r(R_{i,\text{out}}, R_{j,\text{out}}) \frac{\partial R}{\partial R_{j,\text{out}}} u(R_{j,\text{out}}) \\
 &- 2 \sum_{i=1}^3 \sum_{j=1}^3 \frac{\partial R}{\partial R_{i,\text{in}}} u(R_{i,\text{in}}) r(R_{i,\text{in}}, R_{j,\text{out}}) \frac{\partial R}{\partial R_{j,\text{out}}} u(R_{j,\text{out}}) \quad (8)
 \end{aligned}$$

The correlation coefficients are given by

$$r(R_{i,\text{in}}, R_{j,\text{in}}) = r(R_{i,\text{out}}, R_{j,\text{out}}) = r(R_{i,\text{in}}, R_{j,\text{out}}) = 0 \quad (9a)$$

for  $i \neq j$  and

$$r(R_{i,\text{in}}, R_{i,\text{out}}) = 1 \quad (9b)$$

for the correlated contributions to the uncertainty. For the uncorrelated uncertainty contributions, all correlation coefficients are 0. Equation 8 therefore simplifies to

$$\begin{aligned}
 u^2(\Delta R) &= \sum_{i=1}^3 \left( \frac{\partial R}{\partial R_{i,\text{in}}} u(R_{i,\text{in}}) \right)^2 + \sum_{i=1}^3 \left( \frac{\partial R}{\partial R_{i,\text{out}}} u(R_{i,\text{out}}) \right)^2 \\
 &- 2 \sum_{i=1}^3 \frac{\partial R}{\partial R_{i,\text{in}}} u(R_{i,\text{in}}) r(R_{i,\text{in}}, R_{i,\text{out}}) \frac{\partial R}{\partial R_{i,\text{out}}} u(R_{i,\text{out}}). \quad (10)
 \end{aligned}$$

Using  $u_c(R)$  for the correlated and  $u_u(R)$  for the uncorrelated uncertainty contribution, the formula becomes more obvious:

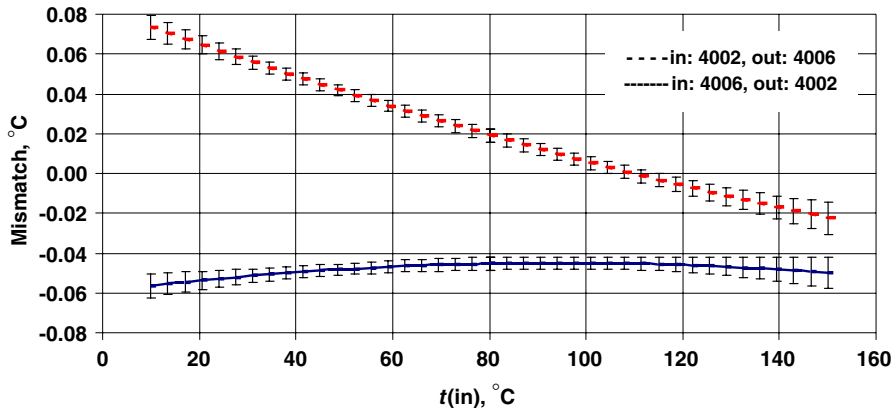
$$\begin{aligned}
 u^2(\Delta R) &= \sum_{i=1}^3 \left( \frac{\partial R}{\partial R_{i,\text{in}}} u_c(R_{i,\text{in}}) \right)^2 + \sum_{i=1}^3 \left( \frac{\partial R}{\partial R_{i,\text{out}}} u_c(R_{i,\text{out}}) \right)^2 \\
 &+ \sum_{i=1}^3 \left( \left( \frac{\partial R}{\partial R_{i,\text{in}}} - \frac{\partial R}{\partial R_{i,\text{out}}} \right) u_u(R_i) \right)^2. \quad (11)
 \end{aligned}$$

As can be seen from Table 5, Eq. 11 is a reasonable approximation. Because  $t_{\text{in}} \neq t_{\text{out}}$ , the terms for correlated contributions do not cancel, but still are considerably reduced.

The results of the calculations are shown in Fig. 2. The uncertainty of the calibration is still much smaller than the mismatch of the two sensors. Nevertheless, larger uncertainties may drastically reduce the number of matched pairs that can be found in a batch of sensors. It is also clear from Fig. 2 that an exchange of the sensors for

**Table 5** Uncertainties calculated with different methods

	$t(\text{in}) = 140^\circ\text{C}$ $t(\text{out}) = 130^\circ\text{C}$	$t(\text{in}) = 110^\circ\text{C}$ $t(\text{out}) = 90^\circ\text{C}$	$t(\text{in}) = 80^\circ\text{C}$ $t(\text{out}) = 70^\circ\text{C}$	$t(\text{in}) = 45^\circ\text{C}$ $t(\text{out}) = 35^\circ\text{C}$
Analytical method, Eq. 11	5.60 mK	3.20 mK	3.15 mK	2.92 mK
MCM	5.20 mK	3.11 mK	3.19 mK	2.64 mK



**Fig. 2** Mismatch of the two temperature sensors and the uncertainty associated with the temperature difference. Both are calculated for the same pair of sensors, but the sensors for incoming and outgoing water have been exchanged

the incoming and the outgoing water leads to a considerable change in the matching error.

#### 4 Monte Carlo Method for the Calculation of Uncertainties

Working Group 1 (WG1) of the Joint Committee for Guides in Metrology (JCGM) has taken over responsibility for the *Guide to the Expression of Uncertainty in Measurement* (GUM) [4] from ISO TAG 4. The task of WG1 is to promote the use of the GUM and to prepare supplementary guides for its broad application [5]. Supplement 1 is expected to be published soon. It is based on the general concept of ‘propagating PDFs’ to obtain the probability distribution function (PDF) for the measurand, and it recommends a Monte Carlo method (MCM) as a basic tool for this purpose [5]. The propagation of PDFs using an MCM is straightforward, and its use is suggested for cases where the model for evaluating the uncertainty is strongly non-linear or highly complicated such that the standard framework of the GUM is not very practical [5,6].

The basic idea of a MCM is to generate a representative set of samples of possible values of the output quantities and sort them to a frequency distribution. As the sample size increases, this frequency distribution becomes a numerical representation of the PDF for the output quantities. Possible values of the output quantities are obtained by combining possible values of the input quantities according to the model of the measurement [6].

Since the model for measuring the temperature difference measured with an imperfect pair of Pt-100 sensors is non-linear and complicated, MCM is ideally suited. To that end, the calibration of these sensors is fully simulated by generating possible values of the Pt-100 coefficients, taking *simultaneous* account of the correlated uncertainties due to the calibration temperatures and the electrical measurements. This means that the values of temperature *and* resistance in Eq. 4 are replaced by a set of possible values  $(\tau_1, \tau_2, \tau_3)$  for the temperatures and  $(\rho_1, \rho_2, \rho_3)$  for the resistances. Possible values of temperatures are given by the sum of the corresponding indicated value and the values drawn from the PDFs for the homogeneity and the stability of the bath, the reference standard, and the heat conduction, i.e.  $T_{Ho}$ ,  $T_S$ ,  $T_{RS}$ , and  $T_{HC}$ , multiplied by the corresponding associated uncertainty as given in Table 2. The same holds analogously for the possible values of resistance. This procedure is exemplified by Eq. 12 for the sensor for the incoming water:

$$\begin{aligned} \tau_{i,in} &= t_{i,in} + u_{HO,i} \rho_{R,HO,i,in} + u_{S,i} \rho_{R,S,i,in} + u_{HC,i} \rho_{G,RS} + u_{HC,i} \rho_{R,HC} \\ \rho_{i,in} &= r_{i,in} + u_{T,i} \rho_{R,T,i,in} + u_{EM,i} \rho_{G,EM}, \end{aligned} \quad (12)$$

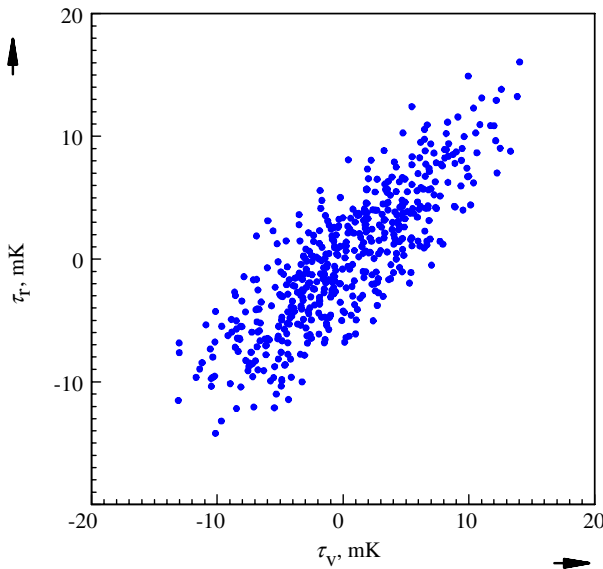
where the subscript  $i \in [1, 3]$  denotes the calibration temperatures and the subscript ‘in’ the ‘incoming’ water. The indicated temperatures and resistances (readings) are  $t_{i,in}$  and  $r_{i,in}$ , respectively. The subscripts of the uncertainties  $u$  and the samples  $\rho$  are explained in Table 2. For instance,  $u_{Ho}$  is the uncertainty due to the homogeneity of the bath temperature and  $\rho_{R,HO,i,in}$  is a sample from a rectangular PDF. Correlation, indicated by ‘c’ in the sixth row of Table 2, is considered by taking the *same* sample from the corresponding PDF, e.g.,  $\rho_{G,RS}$ .

Any such set of possible values of temperatures *and* resistances is then used to compute a set of possible values of the Pt-100 coefficients by matrix inversion, see Eq. 4, for both sensors, ‘in’ and ‘out.’ These sets of values are then used to simulate possible values of the resistance induced by the temperature “seen” by the application, again considering all relevant uncertainties. These resistances are then converted to temperatures using standard Pt-100 coefficients and again taking account of uncertainty contributions due to the electrical measurement. The difference of any such simulated pair of indicated temperatures is sorted in a frequency distribution that, as mentioned above, tends with increasing sample number asymptotically to the PDF for the temperature difference. Using standard statistical tools, one can also compute the expectation and standard deviation that, according to the GUM, are the best estimates of the value of the temperature difference and the uncertainty associated with that value, respectively. Furthermore, one can compute the position and width of requested coverage intervals.

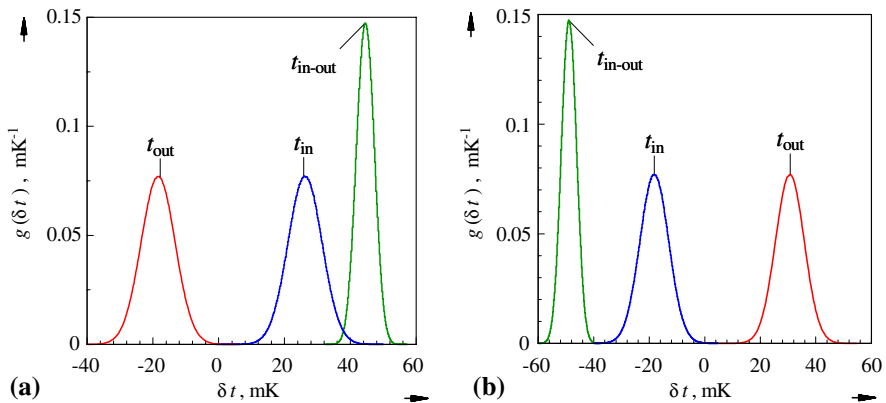
The main advantage of the MCM in this application is that the physics can be simulated concisely, without any approximations. Hence, the MCM is perfectly suited to check analytical solutions, such as shown in Sect. 3, which resort necessarily to some approximations but have the advantage of better acceptance.

It is clear that the MCM leads to the same mismatch of the sensors (see Fig. 2) as the analytical method, but the MCM gives additional information. Figure 3 shows the temperature deviations for sensor pairs that are calibrated simultaneously. One





**Fig. 3** Monte Carlo simulated pair of values of  $\delta t_{in}$  and  $\delta t_{out}$  for  $t_{in} = 45^\circ\text{C}$  and  $t_{out} = 35^\circ\text{C}$  using standard Pt-100 coefficients according to IEC 60751



**Fig. 4** PDFs for the temperatures of both sensors and for the temperature differences.  $\delta t$  indicates the difference with respect to the correct temperatures. Calculations were made for  $t_{in} = 45^\circ\text{C}$  and  $t_{out} = 35^\circ\text{C}$ . (b) was calculated for the same parameters as (a), but the sensors for incoming and outgoing water have been exchanged

recognizes that the possible values of the temperatures are strongly correlated. Sources of this correlation are the use of the same temperature reference standards at all temperatures and for all sensors, the losses due to heat conduction, and the use of the same device for all electrical measurements.

The correlation is even more obvious from Fig. 4, which shows the PDFs for the uncertainty of each of the sensors and the PDF for the uncertainty of the matched pair for the measurement of temperature differences. It becomes clear that taking account

of the correlations reduces the uncertainty considerably when measuring temperature differences. The PDFs are not centered at the correct temperatures because it is assumed that the standard Pt-100 coefficients from IEC 60751 are used in the calculator, which of course are incorrect for the specific sensors.

A comparison of Figs. 4 and 2 shows that, at least for the parameters used in Fig. 4, the analytical method and the MCM lead to basically the same results. For the parameters given in Tables 2 and 3, the uncertainty calculated using the MCM and analytical techniques are compared in Table 5. The agreement is as good as could be expected considering the approximations used for the correlation coefficients in the analytical calculations.

From the data shown in Fig. 4, the correlation coefficient for the uncertainty of the paired sensors can be calculated. The correlation coefficient is about 0.80 and more or less independent of  $\Delta t$  and  $t(\text{in})$ , if the uncertainty contributions given in Table 2 are used. Correlation coefficients for other sets of uncertainty contributions can easily be calculated.

## 5 Conclusion

For the difficult case of paired sensors calibrated with the same equipment and used for the measurement of temperature, it has been shown that analytical methods and MCM lead to basically the same resulting uncertainties. Additionally, MCM allows the calculation of interesting details, such as the correlation coefficient for the measured temperatures or a variance analysis, as a basis for the optimization of the measurement. However, the most important feature of the MCM is the capability to model the underlying physics correctly without any approximations.

It has also become clear that MCM is not limited to scientific applications, but can be very useful in an industrial application like heat meters, where the correct treatment of correlations reduces the resulting uncertainties significantly.

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